Three neutrino Δm^2 scales and the singular seesaw mechanism

E. J. Chun*
Korea Institute for Advanced Study, 207-43 Cheongryangri-dong, Dongdaemun-gu, Seoul 130-012, Korea

C. W. Kim†
Korea Institute for Advanced Study, 207-43 Cheongryangri-dong, Dongdaemun-gu, Seoul 130-012, Korea and Department of Physics & Astronomy, The Johns Hopkins University, Baltimore, Maryland 21218

U. W. Lee‡
Department of Physics, Mokpo National University, Chonnam 534-729, Korea

(Received 2 February 1998; published 21 September 1998)

It is shown that the singular seesaw mechanism can simultaneously explain all the existing data supporting nonzero neutrino masses and mixing. The three mass-squared differences that are needed to accommodate the atmospheric neutrino data (through $\nu_\mu - \nu_s$ oscillation), the solar neutrino data via the MSW mechanism (through $\nu_e - \nu_\tau$ oscillation), and the positive result of $\nu_\mu - \nu_e$ oscillation from the LSND can be generated by this mechanism, whereas the vacuum oscillation solution to the solar neutrino problem is disfavored. We find that the electron and tau neutrino masses are of the order of 10^{-3} eV, and the muon neutrino and a sterile neutrino are almost maximally mixed to give a mass of the order of 1 eV. Two heavy sterile neutrinos have a mass of the order of 1 keV which can be obtained by the double seesaw mechanism with an intermediate mass scale $\sim 10^5$ GeV. A possible origin of such a scale is discussed. [S0556-2821(98)04917-0]

PACS number(s): 14.60.Pq, 14.60.St

I. INTRODUCTION

There are several neutrino oscillation experiments which have indicated the nonzero neutrino masses and mixing. The solar neutrino problem is the first to be noted. The deficit of the solar neutrinos predicted by the standard solar model (SSM) [1] can be explained by the neutrino oscillation between ν_e and ν_x. The ν_x can be ν_μ, ν_τ, or a sterile neutrino. In the case of resonant Mikheyev-Smirnov-Wolfenstein (MSW) transitions [2], it was found [3] that the oscillation parameters Δm^2 and $\sin^2 2\vartheta$ (ϑ is the mixing angle), given by

$$3\times 10^{-6} \lesssim \Delta m^2_{\rm solar}({\rm eV}^2) \lesssim 1.2\times 10^{-5},$$
$$4\times 10^{-3} \lesssim \sin^2 2\vartheta_{ex} \lesssim 1.2\times 10^{-2}, \tag{1}$$

can explain the solar neutrino problem. The solar neutrino problem can also be solved by invoking vacuum neutrino oscillations [4], in which case the neutrino mass-squared difference is about $\Delta m^2 \sim 10^{-10}$ eV^2.

Another hint of the neutrino masses and mixing comes from experiments on atmospheric neutrinos. Indications in favor of $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_x$ oscillations ($x \neq \mu$) have been found at the Kamiokande [5], IMB [6], and recent Super-Kamiokande [7] and Soudan-II [8] atmospheric neutrino experiments. From the analysis of the Super-Kamiokande and the other data the allowed ranges for the oscillation parameters were obtained [7,9], assuming $\nu_\mu \leftrightarrow \nu_\tau$:

$$3\times 10^{-4} \lesssim \Delta m^2_{\rm atm}({\rm eV}^2) \lesssim 7\times 10^{-3}$$
$$0.8 \lesssim \sin^2 2\vartheta_{\mu\tau} \lesssim 1. \tag{2}$$

The recent results from the CHOOZ reactor experiment appear to exclude the $\nu_\mu \leftrightarrow \nu_e$ oscillation as a solution to the atmospheric neutrino problem [10,7]. The neutrino oscillations $\nu_\mu \leftrightarrow \nu_\tau$ and $\nu_\mu \leftrightarrow \nu_s$ give rise to a similar result for the atmospheric neutrino anomaly since ν_s and ν_τ are not distinguishable in the current experiments. New experiments with high statistics at Super-Kamiokande and the Imaging of Cosmic and Rare Underground Signals (ICARUS) experiment can provide the discrimination [11].

Finally, indications in favor of $\overline{\nu}_\mu \to \overline{\nu}_e$ oscillations have been found recently at the Liquid Scintillation Neutrino Detector (LSND) experiment [12], in which antineutrinos originating from the decays of μ^+'s at rest were detected. The KARMEN experiment [13] will be able to cross-check the positive result of LSND in the near future. From the analysis of the data of the LSND experiment and the negative results of other short-base-line experiments (in particular, the Bugey [14] and BNL E776 [15] experiments), one obtains the oscillation parameters:

$$0.3 \lesssim \Delta m^2_{\rm LSND}({\rm eV}^2) \lesssim 2.2 \ \ {\rm eV}^2$$
$$10^{-3} \lesssim \sin^2 2\vartheta_{e\mu} \lesssim 4\times 10^{-2}. \tag{3}$$

All the existing neutrino experiments which are in favor of neutrino oscillations can not be accommodated by a scheme with mixing of ordinary three neutrinos. That is, the allowed ranges of the mass-squared differences which explain the solar neutrino, atmospheric neutrino, and LSND neutrino experiments do not overlap at all. In order to obtain three in-

*Email address: ejchun@kiasph.kaist.ac.kr
†Email address: cwkim@kias.kaist.ac.kr
‡Email address: uwlee@chungkye.mokpo.ac.kr

dependent mass-squared differences which can explain the known three experiments, we need at least four massive neutrinos [16–20]. Furthermore, only two schemes of neutrino mass-squared differences are compatible with the results of all the experiments [19]. Four neutrino masses are divided into two pairs of almost degenerate masses separated by a gap of ~ 1 eV which is indicated by the result of the LSND experiments:

$$
\begin{aligned}
&(\mathrm{A}) \qquad \underbrace{\overbrace{m_1 < m_2}^{\text{atm}} \ll \overbrace{m_3 < m_4}^{\text{solar}}}_{\text{LSND}},\\
&(\mathrm{B}) \qquad \underbrace{\overbrace{m_1 < m_2}^{\text{solar}} \ll \overbrace{m_3 < m_4}^{\text{atm}}}_{\text{LSND}}.
\end{aligned}
\tag{4}
$$

In (A), Δm_{21}^2 is relevant for the explanation of the atmospheric neutrino anomaly and Δm_{43}^2 is relevant for the suppression of solar ν_e's. In (B), the roles of Δm_{21}^2 and Δm_{43}^2 are interchanged.

If there exists a fourth neutrino, it has to be a sterile neutrino as indicated by the invisible decay width of Z^0. The mixing between a fourth neutrino and active neutrinos could be constrained by big-bang nucleosynthesis. The active-sterile neutrino mixings suggested by the known neutrino experiments may increase the effective number of neutrinos and deplete the electron neutrino and antineutrino populations in the nucleosynthesis era, and thus alter significantly the prediction for the primordial ^{4}He mass fraction. Formerly, the effective number of neutrino species was considered to be less than 4, and therefore the large mixings between a sterile neutrino and the active neutrinos solving the solar neutrino and the atmospheric neutrino problem were disfavored [21]. However, because of the recent observational determinations of the primordial deuterium abundance [22,23], the big-bang nucleosynthesis constraints on the nonstandard neutrinos have been revised. It has been argued that an effective number of neutrino species more than 4 (but less than 5) can be acceptable [24], in which case there is room to bring one sterile neutrino species into equilibrium with the known neutrinos in the early universe and therefore no constraints on active-sterile neutrino oscillation parameters can be drawn. More interestingly, it is possible to reconcile sterile neutrinos with big-bang nucleosynthesis even if the effective number of neutrino species turns out to be less than 3. It has been found that the active-sterile neutrino oscillation solving the solar or atmospheric neutrino problem does not increase the effective number of neutrinos if the relic neutrino asymmetry is large enough ($L_\nu > 10^{-5}$) [25].

In the conventional seesaw mechanism, right-handed neutrinos are heavy and make active neutrinos very light. In this paper, we explore the possibility that a right-handed neutrino remains light: that is, the right-handed neutrino mass matrix is singular and has rank 2 in a three-generation model. This is called the singular seesaw mechanism [26] which will be analyzed in Sec. II. Three mass-squared differences for the light four neutrinos will be parametrized in terms of two mass scales: the Dirac mass and Majorana mass of heavy right-handed neutrinos. By construction, the sterile neutrino (a light right-handed neutrino) and an active neutrino are maximally mixed, which is desirable for the solution of the atmospheric neutrino anomaly. In Sec. III, we will find the region of the two mass parameters by which all the neutrino data mentioned above can be accommodated. It is then required that the Dirac mass scale is ~ 1 eV and the heavy Majorana mass scale is ~ 1 keV. Such low mass scales are shown to imply the presence of an intermediate scale $\sim 10^5$ GeV and the grand unification scale $\sim 10^{16}$ GeV in the double seesaw mechanism introduced in Sec. IV. Finally, we conclude in Sec. V.

II. SINGULAR SEESAW MECHANISM

In the singular seesaw mechanism [26], the neutrino mass matrix is written by

$$\mathcal{M} = \begin{bmatrix} 0 & \epsilon M_D \\ \epsilon M_D^\dagger & M_M \end{bmatrix}, \tag{5}$$

where M_D is the usual Dirac neutrino mass matrix and M_M is the right-handed Majorana neutrino mass matrix taken to be a singular (rank-2) 3×3 matrix. Here, we assume that there is no hierarchical structure in the mass matrices M_D and M_M whose elements are of the same order of magnitude, denoted by M. The small number ϵ encodes the hierarchical structure of the Dirac and sterile (right-handed) neutrino masses. The value M is related to the physics of lepton number violation and new physics. Two parameters ϵ and M are to be determined later. We write the neutrino states in the interaction basis as

$$\Psi_I = \begin{bmatrix} \psi_{(e,\mu,\tau),l} \\ \psi_{(e,\mu,\tau),S} \end{bmatrix}, \tag{6}$$

where $\psi_{(e,\mu,\tau),l}$ and $\psi_{(e,\mu,\tau),S}$ represent, respectively, the three standard active and sterile neutrinos. In the context of the singular seesaw mechanism, a combination of $\psi_{(e,\mu,\tau),S}$ becomes light. In order to obtain physical neutrino states and mass eigenvalues, we perform diagonalization by several steps. The first step is to diagonalize the Majorana part by a rotation matrix R such as

$$\mathcal{M} = \begin{bmatrix} 1 & 0 \\ 0 & R^T \end{bmatrix} \begin{bmatrix} 0 & \epsilon M_D R^T \\ \epsilon R M_D^\dagger & \tilde{M}_M \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & R \end{bmatrix}, \tag{7}$$

where

$$\tilde{M}_M = R M_M R^T = \text{diagonal matrix}. \tag{8}$$

Since M_M is a rank-2 singular matrix, we can take zero for the 11-element of the diagonal matrix $\tilde{M}_M$. Then, we rewrite the mass matrix as

$$\begin{bmatrix} 0 & \epsilon M_D R^T \\ \epsilon R M_D^\dagger & \tilde{M}_M \end{bmatrix} = \begin{bmatrix} \epsilon M_\alpha & \epsilon M_\beta \\ \epsilon M_\beta^\dagger & \tilde{M}_m \end{bmatrix}, \tag{9}$$

where M_α is a 4×4 matrix, M_β is a 4×2 matrix, and $\tilde{M}_m$ is a 2×2 diagonal matrix. In particular, the matrix elements $M_{\alpha ij}$ for $i,j=1,2,3$ are zero. The values of the other matrix elements can be taken to be arbitrary. The next step is to block-diagonalize this mass matrix as follows:

$$\begin{bmatrix} \epsilon M_\alpha & \epsilon M_\beta \\ \epsilon M_\beta^\dagger & \tilde{M}_m \end{bmatrix} = \begin{bmatrix} 1 & \epsilon P \\ -\epsilon P^T & 1 \end{bmatrix} \begin{bmatrix} Q & 0 \\ 0 & \tilde{M}_m \end{bmatrix} \begin{bmatrix} 1 & -\epsilon P \\ \epsilon P^T & 1 \end{bmatrix}. \tag{10}$$

Here the 4×2 matrix P and the light neutrino mass matrix Q are given by

$$P = M_\beta \tilde{M}_m^{-1},$$

$$Q = \epsilon M_\alpha - \epsilon^2 P \tilde{M}_m P^T = \epsilon M_\alpha - \epsilon^2 M_\beta \tilde{M}_m^{-1} M_\beta^T. \tag{11}$$

Finally, the light neutrino mass matrix Q is diagonalized by a 4×4 unitary rotation matrix U. The mass eigenstates (physical states) are given by

$$\Psi_P = \begin{bmatrix} U & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\epsilon P \\ \epsilon P^T & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} \psi_{(e,\mu,\tau),l} \\ \psi_{(e,\mu,\tau),S} \end{bmatrix}. \tag{12}$$

Let us now determine the masses of six physical neutrinos. The mass matrix relevant for heavy neutrinos is $\tilde{M}_m$. The nonzero values of the matrix elements are of order M. And the physical neutrino fields are given by

$$\nu_5 = \sum_{\alpha=e,\mu,\tau} \epsilon (P_{\alpha,1}\nu_{\alpha,l} + P_{4,1}R_{1,\alpha}\nu_{\alpha,S}) + R_{2,\alpha}\nu_{\alpha,S} \simeq R_{2,\alpha}\nu_{\alpha,S},$$

$$\nu_6 = \sum_{\alpha=e,\mu,\tau} \epsilon (P_{\alpha,2}\nu_{\alpha,l} + P_{4,2}R_{1,\alpha}\nu_{\alpha,S}) + R_{3,\alpha}\nu_{\alpha,S} \simeq R_{3,\alpha}\nu_{\alpha,S}. \tag{13}$$

The masses of the light neutrinos come from diagonalizing the mass matrix Q given by

$$Q = \epsilon M_\alpha - \epsilon^2 M_\beta \tilde{M}_m^{-1} M_\beta^T. \tag{14}$$

Neglecting the ϵ^2 term, the most general matrix is

$$Q = \epsilon M_\alpha = \begin{bmatrix} 0 & 0 & 0 & a \\ 0 & 0 & 0 & b \\ 0 & 0 & 0 & c \\ a & b & c & 0 \end{bmatrix}. \tag{15}$$

The eigenvalues of Q are two zeros and $\pm\sqrt{a^2+b^2+c^2}$. These twofold degeneracies are lifted by the ϵ^2 term $-\epsilon^2 M_\beta \tilde{M}_m^{-1} M_\beta^T$. Therefore, the light neutrinos are, in general, two with mass $\epsilon^2 M$ and two with mass ϵM, their mass difference being of order $\epsilon^2 M$.

When we neglect the ϵ^2 term in the mass matrix, the neutrino mass eigenstates are

$$\nu_{3,4} = \frac{1}{\sqrt{2}} \left(\frac{a\nu_e + b\nu_\mu + c\nu_\tau}{\sqrt{a^2+b^2+c^2}} \pm \nu_S \right), \tag{16}$$

where $\nu_S = R_{1\alpha}\nu_{\alpha,S}$ is the massless component of the sterile neutrino mass matrix. The other two neutrino states (whose masses are of order $\epsilon^2 M$) are orthogonal combinations of $\nu_{3,4}$. Including the ϵ^2 term in the light neutrino mass matrix Q, it will give an order of ϵ to the neutrino mixing. Therefore, we know that the lightest pair of the neutrinos with mass $\epsilon^2 M$ are most likely active neutrinos. However, the compositions of the ν_3 and ν_4 are dependent on the form of the mass matrix. For example, let $a=b=c$; then the electron, muon, and tau neutrino component in $\nu_{3,4}$ is 1/6, respectively. The other half of these neutrinos is the sterile neutrino. As another example, consider $c=0$ and $a=b$; then the electron and muon neutrino component in $\nu_{3,4}$ is 1/4, respectively, and the other half of $\nu_{3,4}$ is sterile.

We summarize the neutrino mass eigenvalues and their mass-squared differences which are relevant to the known neutrino experiments for our discussions. The neutrino masses are determined as follows:

$$m_{\nu_1} \simeq m_{\nu_2} \simeq \epsilon^2 M,$$

$$m_{\nu_3} \simeq m_{\nu_4} \simeq \epsilon M,$$

$$m_{\nu_5} \simeq m_{\nu_6} \simeq M. \tag{17}$$

The two lightest neutrinos ν_1 and ν_2 are mostly active neutrinos. The two medium-weighted neutrinos ν_3 and ν_4 are almost equal combinations of active and sterile neutrinos. The two heaviest neutrinos ν_5 and ν_6 are almost sterile neutrinos. Therefore, the mass-squared differences are given by

$$\Delta m_{21}^2 = (m_{\nu_2}+m_{\nu_1})(m_{\nu_2}-m_{\nu_1}) \simeq \epsilon^4 M^2,$$

$$\Delta m_{43}^2 = (m_{\nu_4}+m_{\nu_3})(m_{\nu_4}-m_{\nu_3}) \simeq \epsilon^3 M^2,$$

$$\Delta m_{42}^2 = (m_{\nu_4}+m_{\nu_2})(m_{\nu_4}-m_{\nu_2}) \simeq \epsilon^2 M^2, \tag{18}$$

and $\Delta m_{42}^2 \simeq \Delta m_{41}^2 \simeq \Delta m_{32}^2 \simeq \Delta m_{31}^2$.

III. DETERMINATION OF ϵ, M, AND THE NEUTRINO MASSES

As shown above, the singular seesaw mechanism has three Δm^2 scales which provide the possibility of explaining

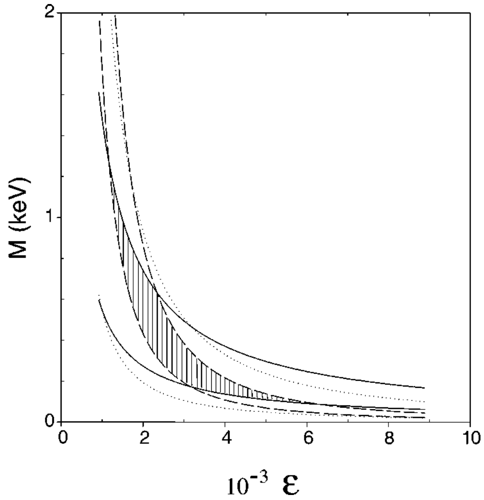

FIG. 1. The regions between the dashed lines, the dotted lines, and the solid lines are allowed by the solar neutrino data, the atmospheric neutrino data, and the LSND data, resepectively. The shaded region accomodates all the three neutrino experiments.

the three known experiments. It is, however, *a priori* uncertain whether the three experimental data (1), (2), and (3) can be accommodated simultaneously since the three Δm^2 are parametrized by only two numbers ϵ and M. We wish to see if the scheme (B) in Eq. (4) can indeed be realized. Note first that the large mixing required for the atmospheric neutrino oscillation is built in the singular seesaw mechanism. That is, a combination of active neutrinos and a sterile neutrino has the maximal mixing to yield degenerate neutrinos ν_3, ν_4. Therefore, the atmospheric neutrino problem can be explained by the large mixing $\nu_\mu \leftrightarrow \nu_s$ oscillation with the mass-squared difference $\Delta m^2_{43} \simeq \epsilon^3 M^2$. The solar neutrino problem is then to be solved by the $\nu_e \leftrightarrow \nu_\tau$ oscillation with the smallest mass-squared difference $\Delta m^2_{21} \simeq \epsilon^4 M^2$. As a consequence, the $\nu_\mu \leftrightarrow \nu_e$ oscillation occurs automatically with the largest mass-squared difference $\Delta m^2_{42} \simeq \epsilon^2 M^2$, which may yield observable signals in the $\nu_\mu \leftrightarrow \nu_e$ oscillation experiments.

Let us now look for the parameter region of (ϵ, M) determined by the neutrino experiments, that is, $\epsilon^4 M^2 = \Delta m^2_{\rm solar}$, $\epsilon^3 M^2 = \Delta m^2_{\rm atm}$, and/or $\epsilon^2 M^2 = \Delta m^2_{\rm LSND}$. From the first two identities based on the MSW solution to the solar neutrino problem and the atmospheric neutrino oscillation, one finds the region of a crescent shape inside $\epsilon = (0.43-9.6) \times 10^{-3}$ and $M = (0.02-9.3)$ keV. Remarkably, this region gives rise to $\Delta m^2_{42} = \epsilon^2 M^2$ in the sensitivity range of the LSND and KARMEN experiments. Imposing the LSND positive result, the allowed region of (ϵ, M) is further reduced and, in fact, determined solely by the solar neutrino and LSND data. The common region of (ϵ, M) which can explain all three known experiments lies inside

$$\epsilon = (1.1-6.4) \times 10^{-3}, \quad M = (0.086-1.3) \text{ keV}, \quad (19)$$

which is shown in Fig. 1. From Eqs. (17) and (19), we find the following typical values for the neutrino masses:

$$m_{\nu_{1,2}} \sim 10^{-3} \text{ eV},$$

$$m_{\nu_{3,4}} \sim 1 \text{ eV},$$

$$m_{\nu_{5,6}} \sim 1 \text{ keV}. \quad (20)$$

If the solar neutrino deficit is due to the vacuum oscillation, the solar neutrino data and the atmospheric neutrino data yield $\epsilon = \Delta m^2_{21}/\Delta m^2_{43} \lesssim 3 \times 10^{-7}$. Then the heavy right-handed neutrinos $(\nu_{5,6})$ get mass 100 MeV$\lesssim M \lesssim$50 GeV and 30 eV$\lesssim m_{\nu_{\mu,s}} \lesssim$700 eV. The right-handed neutrinos with such high masses overclose the universe as they cannot decay fast into the standard model particles. The muon and light sterile neutrinos can be candidates for hot dark matter, satisfying the overclosure bound $\Sigma_i m_{\nu_i} \lesssim 94 h^2 \Omega_\nu$ eV. But they are too heavy to provide a good fit for the structure formation as $\Omega_\nu \lesssim 0.3$ is required in mixed dark matter models [27,28]. Furthermore, one finds no region of (ϵ, M) which accommodates all the three neutrino experiments. Therefore, our model disfavors the possibility of solving the solar neutrino problem in terms of the vacuum oscillation. Thus, the existence of hot dark matter desirable for structure formation in the mixed dark matter scenario is a natural consequence of the singular seesaw mechanism under consideration.

The neutrino components $\nu_{5,6}$ being sterile decouple at a very high temperature (say, $T_s > 100$ GeV), and their abundance compared to active neutrinos is suppressed by a factor of $n_s/n_a = g_{eff}(T_a)/g_{eff}(T_s)$ where $n_{s,a}$ are the relic number densities of sterile and active neutrinos, respectively, and g_{eff} is the effective number of relativistic degrees of freedom at the respective decoupling temperature. Recalling $T_a \sim 1$ MeV, the abundance of sterile neutrinos is smaller than that of active neutrinos roughly by factor of 10 [29]. Now that the mixing between the heavy sterile and active neutrinos ($\tan\theta \approx \epsilon$) is small enough, the heavy sterile neutrinos can never be brought into thermal equilibrium [21]. Therefore, it follows from Fig. 1 that the heavy sterile neutrino can be a candidate for the warm dark matter in some parameter range.

IV. DISCUSSIONS ON THE MASS SCALE OF THE STERILE NEUTRINOS

It seems unnatural to have sterile (right-handed) neutrinos with mass scale $M \approx 1$ keV. Such a low mass scale of lepton number violation is not acceptable. In the usual seesaw mechanism this scale is about 10^{12} GeV or grand unified theory (GUT) scale. In order to raise the lepton number violation scale in our scheme, we introduce the ''double seesaw mechanism'' in which extra sterile neutrinos are needed in addition to the ordinary right-handed neutrinos. A simple realization of the double seesaw mechanism can be found in GUT with intermediate step breakings. The minimal (nonsupersymmetric) grand unification model is ruled out from the study of the gauge coupling evolution [30], but nonminimal GUT models and supersymmeric GUT models can successfully accommodate the gauge coupling unification and the absence of proton decay. As an example, let us consider a GUT model with E_6 unification group. It has five neutral particles in each generation of the fermion **27** representation: Two [**16** under SO(10)] of them are the neutrinos in the discussion: other three neutrinos [**1**+**10** under SO(10)] are heavy neutrinos with GUT scale masses. Suppose that the mass matrix of the active neutrinos, the right-handed neutrinos, and the extra sterile neutrinos takes the form

$$\mathcal{M}=\begin{bmatrix} 0 & 0 & M_L \\ 0 & 0 & M_R \\ M_L^T & M_R^T & M_S \end{bmatrix}. \tag{21}$$

Here M_L is the Dirac mass matrix originating from the electroweak symmetry breaking, and M_R, M_S are generated from a higher symmetry breaking. Note that M_L, M_R are 3×9 matrices, and M_S is a 9×9 matrix in the three-generation model. It should be mentioned that the mass matrix (21) requires fine-tunings which may not be a serious problem in supersymmetric theories. Given the hierarchy $M_L \ll M_R \ll M_S$, the seesaw mechanism with the ultraheavy neutrino masses M_S gives rise to the 6×6 matrix,

$$\begin{aligned}\mathcal{M}_{sub}&=-\begin{bmatrix} M_L \\ M_R \end{bmatrix} M_S^{-1}[M_L^T \quad M_R^T] \\ &=-\begin{bmatrix} M_L M_S^{-1} M_L^T & M_L M_S^{-1} M_R^T \\ M_R M_S^{-1} M_L^T & M_R M_S^{-1} M_R^T \end{bmatrix},\end{aligned} \tag{22}$$

which can be identified with the matrix (5) apart from the upper-left corner. Note that the singular seesaw mechanism requires the lower-right submatrix of Eq. (22) to be singular. The nonzero contribution in the upper-left part of the matrix (22) is of the order of the solar neutrino mass scale $\epsilon^2 M \sim 10^{-3}$ eV which does not alter our conclusion.

Following the discussion in the previous section, we can find typical scales of M_R and M_S. Namely, requiring $\epsilon = M_L/M_R$, $M = M_R^2/M_S$, and $M_L \simeq 100$ GeV, one obtains

$$M_R \approx 10^5 \text{ GeV}, \quad M_S \approx 10^{16} \text{ GeV}. \tag{23}$$

It is worth emphasizing that the heaviest mass scale M_S coincides with the conventional GUT scale. On the other hand, the intermediate scale M_R turns out to be considerably lower than the conventional scale 10^{12} GeV desirable for the usual seesaw mechanism. In GUT models, various intermediate scales of gauge symmetry breaking can be made consistent with the unification of gauge coupling constants. In particular, such a low scale M_R may be obtained by introducing certain exotic particles to the GUT model [31].

The scale M_R may be related to the physics of supersymmetry breaking. In the supersymmetric standard model, supersymmetry breaking can be mediated either by gravitation or by gauge interactions [32]. The latter scheme provides a natural suppression of flavor violation in the supersymmetric sector and yields distinctive phenomenological and cosmological consequences [33,34]. The minimal type of such theories requires the existence of vectorlike quarks and leptons (messengers) at the mass scale (10^4-10^5) GeV. This is a just right scale for M_R under discussion. Therefore, in the supersymmetric standard model with gauge-mediated supersymmetry breaking, it is conceivable that some messengers are vectorlike sterile neutrinos with the mass matrix given in Eq. (21).

V. CONCLUSIONS

Motivated by recent experimental evidence for nonzero neutrino masses and mixing, we have examined the consequences of the singular seesaw mechanism. The three mass-squared scales required for simultaneously explaining the solar and atmospheric neutrino anomalies, and the LSND data, can be realized if the Majorana mass matrix of right-handed neutrinos is rank 2. Without assuming any hierarchies in the Dirac and Majorana mass matrices, three mass-squared values are found to be determined by two mass parameters: the Dirac mass for the active neutrinos and the Majorana mass for heavy right-handed neutrinos. The singular seesaw mechanism cannot accommodate simultaneously the vacuum oscillation explanation of the solar neutrino deficit and the atmospheric neutrino oscillation. However, the MSW solution to the solar neutrino problem is consistent with the model and the existence of the LSND mass scale is also explained. The almost maximal mixing of a sterile neutrino with the muon neutrino (having the Dirac mass $m_{\nu_{3,4}} \sim 1$ eV) explains the atmospheric neutrino anomaly, and the mixing of the electron and tau neutrino explains the solar neutrino anomaly (having the lightest mass $m_{\nu_{1,2}} \sim 10^{-3}$ eV). Two massive right-handed neutrinos turn out to be rather light (having a Majorana mass $m_{\nu_{5,6}} \sim 1$ keV). We stress that the existence of hot dark matter (consists of $\nu_{\mu,s}$) desirable for the structure formation of the universe is a natural consequence of our scheme. In addition, warm dark matter can be provided by the heavy right-handed neutrinos. We have introduced the double seesaw mechanism in which the two low mass scales $m_{\nu_{3,4}}$ and $m_{\nu_{5,6}}$ are generated by the weak scale $M_L \sim 100$ GeV and an intermediate scale $M_R \sim 10^5$ GeV together with the usual grand unification scale $M_S \sim 10^{16}$ GeV. A candidate for the intermediate scale M_R can be found in GUT models with an intermediate step breaking or in gauge-mediated supersymmetry-breaking models.

ACKNOWLEDGMENTS

U.W.L. thanks KIAS for the kind hospitality during his visit. E.J.C. would like to thank Alexei Smirnov for various comments on this work and ICTP for its hospitality during the Extended workshop on Highlights in Astroparticle Physics. E.J.C. is supported by the Non-Directed Research Fund of Korea Research Foundation, 1996.

[1] J. N. Bahcall and M. H. Pinsonneault, Rev. Mod. Phys. **67**, 781 (1995); S. Turck-Chièze *et al.*, Phys. Rep. **230**, 57 (1993); V. Castellani *et al.*, *ibid.* **281**, 309 (1997); A. Dar and G. Shaviv, Nucl. Phys. B (Proc. Suppl.) **48**, 335 (1996).

[2] S. P. Mikheyev and A. Yu. Smirnov, Yad. Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1985)]; Nuovo Cimento C **9**, 17 (1986); L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); **20**, 2634 (1979).

[3] GALLEX Collaboration, Phys. Lett. B **285**, 390 (1992); P. I. Krastev and S. T. Petcov, *ibid.* **299**, 99 (1993); G. Fiorentini *et al.*, Phys. Rev. D **49**, 6298 (1994); N. Hata and P. G. Langacker, *ibid.* **56**, 6107 (1997).

[4] V. Barger, R. J. N. Phillips, and K. Whisnant, Phys. Rev. Lett. **69**, 3135 (1992); P. I. Krastev and S. T. Petcov, *ibid.* **72**, 1960 (1994).

[5] Y. Fukuda *et al.*, Phys. Lett. B **335**, 237 (1994); **388**, 397 (1996).

[6] R. Becker-Szendy *et al.*, Nucl. Phys. B (Proc. Suppl.) **38**, 331 (1995).

[7] Y. Totsuka, presented at the Lepton-Photon '97, Hamburg, Germany, 1997; E. Kearns, presented at the ITP conference on Solar Neutrinos: News About SNUs, Santa Barbara, 1997.

[8] S. M. Kasahara *et al.*, Phys. Rev. D **55**, 5282 (1997); T. Kafka, presented at Taup 97, LNGS, Asssergi, Italy, 1997.

[9] M. C. Gonzalez-Garcia, H. Nunokawa, O. Peres, T. Stanev, and J. W. F. Valle, Phys. Rev. D (to be published), hep-ph/9801368; V. Barger, T. J. Weiler, and K. Whisnant, hep-ph/9712495.

[10] CHOOZ Collaboration, M. Apollonio *et al.*, Phys. Lett. B **420**, 397 (1998).

[11] F. Vissani and A. Yu. Smirnov, Phys. Lett. B **432**, 376 (1998); Q. Y. Liu and A. Yu. Smirnov, Nucl. Phys. B **524**, 505 (1998).

[12] C. Athanassopoulos *et al.*, Phys. Rev. Lett. **77**, 3082 (1996); Report No. UCRHEP-E197, nucl-ex/9709006.

[13] B. Bodmann *et al.*, Nucl. Phys. **A553**, 831c (1993); K. Eitel, presented at 32nd Recontres de Moriond (hep-ex/9706023).

[14] B. Achkar *et al.*, Nucl. Phys. **B434**, 503 (1995).

[15] L. Borodovsky *et al.*, Phys. Rev. Lett. **68**, 274 (1992).

[16] J. T. Peltoniemi and J. W. F. Valle, Nucl. Phys. **B406**, 409 (1993); D. O. Caldwell and R. N. Mohapatra, Phys. Rev. D **50**, 3477 (1994); J. J. Gomez-Cadenas and M. C. Gonzalez-Garcia, Z. Phys. C **71**, 443 (1996); S. Goswami, Phys. Rev. D **55**, 2931 (1997).

[17] E. Ma and P. Roy, Phys. Rev. D **52**, R4780 (1995); E. J. Chun, A. S. Joshipura, and Al. Y. Smirnov, Phys. Lett. B **357**, 608 (1995); Phys. Rev. D **54**, 4654 (1996); R. Foot and R. R. Volkas, *ibid.* **52**, 6595 (1995); Z. Berezhiani and R. N. Mohapatra, *ibid.* **52**, 6607 (1995); E. Ma, Mod. Phys. Lett. A **11**, 1893 (1996); A. Yu. Smirnov and M. Tanimoto, Phys. Rev. D **55**, 1665 (1997).

[18] S. M. Bilenky *et al.*, Phys. Rev. D **54**, 4432 (1996).

[19] S. M. Bilenky, C. Giunti, and W. Grimus, Eur. Phys. J. **61**, 297 (1998).

[20] N. Okada and O. Yasuda, Int. J. Mod. Phys. A **12**, 3669 (1997).

[21] K. Kainulainen, Phys. Lett. B **237**, 440 (1990); R. Barbieri and A. Dolgov, Nucl. Phys. **B349**, 743 (1991); K. Enqvist, K. Kainulainen, and M. Thomson, *ibid.* **B373**, 498 (1992); J. M. Cline, Phys. Rev. Lett. **68**, 3137 (1992); X. Shi, D. Schramm, and B. D. Fields, Phys. Rev. D **48**, 2563 (1993).

[22] A. Songalia, E. J. Wampler, and L. L. Cowie, Nature (London) **385**, 137 (1997); M. Rugers and C. J. Hogan, Astrophys. J. Lett. **459**, L1 (1996); Astron. J. **111**, 2135 (1996); R. F. Carswell *et al.*, Mon. Not. R. Astron. Soc. **268**, L1 (1994); A. Songalia, L. L. Cowie, C. Hogan, and M. Rugers, Nature (London) **368**, 599 (1994).

[23] S. Burles and D. Tytler, astro-ph/9603070; D. Tytler, X. Fan, and D. Tytler, astro-ph/9603069; D. Tytler and X. M. Fan, Bull. Am. Astron. Soc. **26**, 1424 (1994).

[24] C. J. Copi, D. N. Schramm, and M. S. Turner, Phys. Rev. Lett. **75**, 3981 (1995); Phys. Rev. D **55**, 3389 (1997); B. D. Fields and K. A. Olive, Phys. Lett. B **368**, 103 (1996); New Astron. **1**, 77 (1996); C. Y. Cardall and G. M. Fuller, Phys. Rev. D **54**, 1260 (1996); astro-ph/9603071; P. J. Kernan and S. Sarkar, Phys. Rev. D **54**, 3681 (1996); N. Hata, G. Steigman, S. Budman, and P. Langacker, *ibid.* **55**, 540 (1997).

[25] R. Foot and R. R. Volkas, Phys. Rev. Lett. **75**, 4350 (1995).

[26] R. Johnson, S. Ranfone, and J. Schechter, Phys. Lett. B **179**, 355 (1986); Phys. Rev. D **35**, 282 (1987); M. Gronau, R. Johnson, S. Ranfone, and J. Schechter, *ibid.* **37**, 2597 (1988); T. J. Allen, R. Johnson, S. Ranfone, J. Schechter, and J. W. F. Valle, Mod. Phys. Lett. A **6**, 1967 (1991); Sheldon L. Glashow, Phys. Lett. B **256**, 255 (1991).

[27] Q. Shafi and F. Stecker, Phys. Rev. Lett. **53**, 1292 (1984); S. Achilli, F. Occhionero, and R. Scaramella, Astrophys. J. **299**, 577 (1985); S. Ikeuchi, C. Norman, and Y. Zahn, *ibid.* **324**, 33 (1988); A. van Dalen and R. K. Schaefer, *ibid.* **398**, 33 (1992); M. Davis, F. Summers, and D. Schlegel, Nature (London) **359**, 393 (1992); A. Klypin *et al.*, Astrophys. J. **416**, 1 (1993); D. Schlegel *et al.*, *ibid.* **427**, 512 (1994).

[28] J. R. Primack, J. Holtzman, A. Klypin, and D. O. Caldwell, Phys. Rev. Lett. **74**, 2160 (1995).

[29] K. A. Olive and M. S. Turner, Phys. Rev. D **25**, 213 (1982).

[30] C. Giunti, C. W. Kim, and U. W. Lee, Mod. Phys. Lett. A **6**, 1745 (1991); J. Ellis, S. Kelly, and D. V. Nanopoulos, Phys. Lett. B **260**, 131 (1991); U. Amaldi, W. de Boer, and H. Furstenau, *ibid.* **260**, 447 (1991); P. Langaker and M. Luo, Phys. Rev. D **44**, 817 (1991).

[31] See, for example, M. Lindner and M. Weiser, Phys. Lett. B **383**, 405 (1996); R. Foot, *ibid.* **420**, 333 (1998).

[32] M. Dine and A. Nelson, Phys. Rev. D **48**, 1277 (1993); M. Dine, A. Nelson, and Y. Shirman, *ibid.* **51**, 1362 (1995); M. Dine, A. Nelson, Y. Nir, and Y. Shirman, *ibid.* **53**, 2658 (1996).

[33] S. Dimopoulos, M. Dine, S. Raby, and S. Thomas, Phys. Rev. Lett. **76**, 3494 (1996); D. R. Stump, M. Wiest, and C. P. Yuan, Phys. Rev. D **54**, 1936 (1996); S. Ambrosanio, G. L. Kane, G. D. Kribs, S. P. Martin, and S. Mrenna, Phys. Rev. Lett. **76**, 3498 (1996); Phys. Rev. D **54**, 5395 (1996); S. Dimopoulos, S. Thomas, and J. D. Wells, *ibid.* **54**, 3283 (1996); K. S. Babu, C. Kolda, and F. Wilczek, Phys. Rev. Lett. **77**, 3070 (1996); K. Kiers, J. N. Ng, and G. Wu, Phys. Lett. B **381**, 177 (1996); J. L. Lopez and D. V. Nanopoulos, Phys. Rev. D **55**, 4450 (1997); H. Baer, M. Brhlik, C.-h. Chen, and X. Tata, *ibid.* **55**,

4463 (1997); A. Ghosal, A. Kundu, and B. Mukhopadhyaya, *ibid.* **56**, 504 (1997); D. A. Dicus, B. Dutta, and S. Nandi, Phys. Rev. Lett. **78**, 3055 (1997); S. Ambrosanio, G. D. Kribs, and S. P. Martin, Phys. Rev. D **56**, 1761 (1997).

[34] E. J. Chun, H. B. Kim, and J. E. Kim, Phys. Rev. Lett. **72**, 1956 (1994); A. de Gouvea, T. Moroi, and H. Murayama, Phys. Rev. D **56**, 1281 (1997); J. Hashiba, M. Kawasaki, and T. Yanagida, Phys. Rev. Lett. **79**, 4525 (1997).